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The report summarizes results of experimental and theoretical investigations into the nonlinear response and control of structural elements. Methods for the analysis and design of control procedures applicable to certain nonlinear distributed parameter systems were investigated. Analytical and computational techniques were developed for evaluating the nonlinear effects on control designs. Bench-type experiments were conducted for validating some of the theoretical results.

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Final Report on

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Grant No. AFOSR-F49620-87-C-0088

by

A. H. Nayfeh, J. A. Burns, and E. M. Cliff

Virginia Polytechnic Institute and State University

Blacksburg, VA 24061

May 18, 1989

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Abstract

The report summarizes results of experimental and theoretical investigations into the nonlinear response and control of structural elements. Methods for the analysis and design of control procedures applicable to certain nonlinear distributed parameter systems were investigated. Analytical and computational techniques were developed for evaluating the nonlinear effects on control designs. Bench-type experiments were conducted for validating some of the theoretical results.

I. Objectives

The objectives of the program are

(a) to study various issues arising in the design of controllers for a class of nonlinear, infinite-dimensional dynamical systems. (These issues include the use of ouadratic approximations of energy functionals and the effect of interchanging the linearization step and the truncation step in certain design procedures.)

(b) to study and define domains in parameter space in which large, complex, possibly damaging and sensitive-to-initial-conditions motions may occur in the response of multidegree-of-freedom systems with quadratic and cubic nonlinearities to nonstationary (pulses, impacts, wavepackets) and nonideal excitations (i.e., control inputs and unmodeled disturbances);

(c) to develop an experimental program to test various theoretical and computational results, these experiments will test the validity of proposed controller designs and the regions of validity of analytical solutions.

II. Degrees Granted

1. Ph.D. - Lawrence D. Zavodney, 1987 "A Theoretical and Experimental Investigation of Parametrically Excited Nonlinear Mechanical Systems"

2. Ph.D. - Joseph Z. Ben-Asher, 1988 "Time Optimal Slewing of Flexible Space Craft"

3. Ph.D. - Robert E. Miller, 1988 "Approximation of the LQR Control Problem for Systems Governed by partial Functional Differential Equations"

4. Ph.D. - Samir J. Serhan, 1989 "Response of Nonlinear Structures to Deterministic and Random Excitations"

5. Ph.D. - Zhuangyi Liu, 1989 "Approximation and Control of a Thermoviscoelastic System"

6. Ph.D. - David D. Hill, 1989 "Finite Dimensional Approximations of Distributed Parameter Control Systems"

7. Ph.D. - Rajiv S. Chowdhry, 19[illegible] "Optimal Rigid-Body Rotational Maneuvers"

8. Ph.D. - Sungkwon Kang, 1990 "A Control Problem for Burgers' Equation"

9. Ph.D. - P. Frank Pai, 1990 "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams"

10. Ph.D. - Balakumar Balachandran, 1990 "A Theoretical and Experimental Study of Modal Interactions in Structures" in progress

11. Ph.D. - H. Marrekchi, in progress

12. Ph.D. - R. Spies, in progress

13. Ph.D. - M. Tadi, in progress

14. Ph.D. - Marwan Bikdash, in progress

15. Ph.D. - Jafar Hadian, in progress

16. Ph.D. - D. V. De Walt, in progress

17. M.S. - Michael A. Colbert, 1990 "A Real-Time Optical Measurement System"

III. Publications

1. J. Z. Ben-Asher, J. A. Burns, and E. M. Cliff, "Time-Optimal Slewing of a Flexible Spacecraft," Proceedings of the 26th IEEE Conference on Decision and Control, pp. 524-528, 1987; also Journal of Guidance, Control, and Dynamics, to appear.

   The time-optimal slewing problem of flexible spacecraft is considered. The System is discretized by the assumed modes method, and the problem is solved for a linearized model in reduced state space by parameter optimization. Optimality is verified by the Maximum Principle. The linear solution is further used to obtain time-optimal solutions for the nonlinear problem. Some interesting symmetric and asymptotic properties are shown to be possessed by both the linear and the nonlinear problems.

2. J. A. Burns, E. M. Cliff, and R. E. Miller, "Control of a Viscoelastic Shaft with Attached Tip Mass," Proceedings of the 26th IEEE Conference on Decision and Control, pp. 997-1001, 1987.

In this paper we consider an optimal control problem for the torsional vibrations of a viscoelastic shaft with attached tip- mass at the free end of the shaft. The control is an external torque applied to the tip-mass. The continuous sub-system (the shaft) is assumed to be composed of Boltzmann-type material. We apply a combination of finite elements and averaging numerical schemes to compute suboptimal feedback gains for the distributed parameter model.

3. J. A. Burns and G. H. Geichl, "System Radii of Semi-Discretized Hereditary Control Systems," Proceedings of the 27th IEEE Conference on Decision and Control, 1988.

   In this paper we consider several problems concerning the "robustness" of finite dimensional models constructed via semi-discretization of certain infinite dimensional distributed parameter systems. The term "robustness" as used here refers to the preservation of various system properties under perturbations and not to the robustness of a particular control design. Examples will be presented to illustrate the basic ideas.

4. J. A. Burns, Z. Y. Liu, and R. E. Miller, "Control of a Thermoviscoelastic System," Proceedings of the 27th IEEE Conference on Decision and Control, pp. 1249-1252, 1988.

   In this paper we consider the problem of controlling a thermoviscoelastic system. We assume that the elastic bar is composed of a Boltzmann material (i.e. it exhibits a fading memory behavior), and we include the simplest thermodynamic model to investigate the effect of heat transfer on damping. A quadratic cost optimal control problem is formulated and a finite element/averaging scheme is used to solve the problem with and without thermal effects.

5. J. Z. Ben-Asher, J. A. Burns, and E. M. Cliff, "Computational Methods for the Minimum Effort Problem with Applications to Spacecraft Rotational

Maneuvers," Proceedings of the 1989 IEEE Conference on Control and Applications, Paper WA-6-5, pp. 1-7, 1989.

An optimal control problem is considered for finite- dimensional linear time-invariant systems with scalar control. The cost is taken to be the control effort, defined by the maximum amplitude. The optimal control for a normal system is bang-bang. Four alternative computational schemes are presented and illustrated by two examples. The first example is a double-integrator system, and the second is a slewing maneuver of simple flexible spacecraft.

6. J. Z. Ben-Asher and E. M. Cliff, "Soft-Constrained Time Optimal Maneuvering of a Flexible Spacecraft," Proceedings of the 30th Israeli Conference on Aviation and Astronautics, pp. 218-225, 1989.

The optimal maneuvering problem of flexible spacecraft is considered. The cost is a weighted combination of the terminal time and a measure of the control effort. The system is discretized by the assumed-modes method, and then transformed to modal space. Optimal solutions are obtained by the Maximum Principle. A singular perturbation approach is introduced, by which the system is partitioned into a reduced subsystem with 'slow' response, and a residual subsystem with 'fast' response.

7. R. S. Chowdhry, E. M. Cliff, and F. H. Lutze, "On Optimal Rigid Body Motions," Paper AIAA-89-3616-CP, Proceedings of the AIAA Guidance, Navigation and Control Conference, pp. 1560-1569, 1989.

Optimal rigid body angular motions are investigated in the absence of direct control over one of the angular velocity components. A numerical survey of first order necessary conditions for optimality reveals that over a large range of boundary conditions there are, in general, several distinct extremal solutions. A classification in terms of sub-families of extremal sub-families are established. A locus of Darboux points is obtained, and global optimality of extremal solutions is observed in relation to Darboux

points. Local optimality for the candidate minimizers is verified by investigating the second order necessary conditions.

8. A. H. Nayfeh and P. F. Pai, "Non-Linear Non-Planar Parametric Responses of an Inextensional Beam," International Journal of Non-Linear Mechanics, Vol. 24, No. 2, pp. 139-158, 1989.

The nonlinear integro-differential equations of motion for an inextensional beam are used to investigate the plan‹ and nonplanar responses of a fixed-free beam to a principal parametric excitation. The beam is assumed to undergo flexure about two principal axes and torsion. The equations contain cubic nonlinearities due to curvature and inertia. Two uniform beams with rectangular cross sections are considered: one has an aspect ratio near unity, and the other has an aspect ratio near 6.27. In both cases, the beam possesses a one-to-one internal resonance with one of the natural flexural frequencies in one plane being approximately equal to one of the natural flexural frequencies in the second plane. A combination of the Galerkin procedure and the method of multiple scales is used to construct a first-order uniform expansion for the interaction of the two resonant modes, yielding four first-order nonlinear ordinary-differential equations governing the amplitudes and phases of the modes of vibration. The results show that the nonlinear inertia terms produce a softening effect and play a significant role in the planar responses of high-frequency modes. On the other hand, the nonlinear geometric terms produce a hardening effect and dominate the planar responses of low-frequency modes and nonplanar responses for all modes. If the nonlinear geometric terms were not included in the governing equations, then nonplanar responses would not be predicted. For some range of parameters, Hopf bifurcations exist and the response consists of amplitude- and phase-modulated or chaotic motions.

9. A. M. A. Hamdan and A. H. Nayfeh, "Measures of Modal Controllability and Observability for First- and Second-Order Linear Systems," Journal of Guidance, Control, and Dynamics, Vol. 12, No. 3, pp. 421-428, 1989.

For a system described by the triple (A,B,C) where the matrix A has a set of distinct eigenvalues and a well-conditioned modal matrix, we propose measures of modal controllability and observability. The angles between the left eigenvectors of A and the columns of the matrix B are used to propose modal controllability measures and the angles between the rows of the matrix C, and the right eigenvectors of A are used to propose modal observability measures. Gross measures of controllability of a mode from all inputs and its observability in all outputs are also proposed. These measures are related to other measures suggested in the literature. A closed-form relation between the norm of the residue and the proposed measures is given, thus linking the residue to the unobservability or uncontrollability of the mode. We finally show that the proposed measures can be applied directly to second-order models.

10. L. D. Zavodney and A. H. Nayfeh, "The Non-Linear Response of a Slender Beam Carrying a Lumped Mass to a Principal Parametric Excitation: Theory and Experiment," International Journal of Non-Linear Mechanics, Vol. 24, No. 2, pp. 105-125, 1989.

The non-linear response of a slender cantilever beam carrying a lumped mass to a principal parametric base excitation is investigated theoretically and experimentally. The Euler-Bernoulli theory for a slender beam is used to derive the governing non-linear partial differential equation for an arbitrary position of the lumped mass. The non-linear terms arising from inertia, curvature and axial displacement caused by large transverse deflections are retained up to third order. The linear eigenvalues and eigenfunctions are determined. The governing equation is discretized by Galerkin's method, and the coefficients of the temporal equation - comprised of integral representations of the eigenfunctions and their derivatives - are computed using the linear eigenfunctions. The method of multiple scales is used to determine an approximate solution of the temporal equation for the case of a single mode. Experiments were performed don metallic beams and later on composite beams because all of the metallic beams failed prematurely due to the very large response amplitudes. The results of the experiment show very good qualitative agreement with the theory.

11. A. H. Nayfeh and B. Balachandran, "Modal Interactions in Dynamical and Structural Systems," Applied Mechanics Reviews, Vol. 42, No. 11, Part 2, pp. 175-201, 1989.

We review theoretical and experimental studies of the influence of modal interactions on the nonlinear response of harmonically excited structural and dynamical systems. In particular, we discuss the response of pendulums, ships, rings, shells, arches, beam structures, surface waves, and the similarities in the qualitative behavior of these systems. The systems are characterized by quadratic nonlinearities which may lead to two-to-one and combination autoparametric resonances. These resonances give rise to a coupling between the modes involved in the resonance leading to nonlinear periodic, quasi-periodic, and chaotic motions.

12. A. H. Nayfeh, B. Balachandran, M. A. Colbert, and M. A. Nayfeh, "An Experimental Investigation of Complicated Responses of a Two-Degree-of-Freedom Structure," Journal of Applied Mechanics, Paper No. 89-WA/APM-24, 1989.

Recent theoretical studies indicate that whereas large excitation amplitudes are needed to produce chaotic motions in single-degree-of-freedom systems, extremely small excitation levels can produce chaotic motions in multi-degree-of-freedom systems if they posses autoparametric resonances. To verify these results, we conducted an experimental study of the response of a two-degree-of-freedom structure with quadratic nonlinearities and a two-to-one internal resonance to a primary resonant excitation of the second mode. The responses were analyzed using hardware and software developed for performing time-dependent modal decomposition. We observed periodic, quasi-periodic, and chaotic responses, as predicted by theory. Conditions were found under which extremely small excitation levels produced chaotic motions.

13. P. F. Pai and A. H. Nayfeh, "Nonlinear Nonplanar Oscillations of a Cantilever Beam under Lateral Base Excitations," accepted for publication, International Journal of Non-Linear Mechanics.

The nonplanar responses of a cantilevered beam subject to lateral harmonic base-excitation is investigated using two nonlinear coupled integro-differential equations of motion. The equations contain cubic nonlinearities due to curvature and inertia. Two uniform beams with rectangular cross sections are considered: one has an aspect ratio near unity, and the other has an aspect ratio near 6.27. A combination of the Galerkin procedure and the method of multiple scales is used to construct a first-order uniform expansion for the case of a one-to-one internal resonance and a primary resonance. The results show that the nonlinear geometric terms are important for the responses of low-frequency modes because they produce hardening spring effects. On the other hand, the nonlinear inertia terms dominate the responses of high-frequency modes. We also obtain quantitative results for nonplanar motions and investigate their dynamic behavior. For different range of parameters, the nonplanar motions can be steady whirling motions, whirling motions of the beating type, or chaotic motions. Furthermore, we investigate the effects of damping.

14. J. Z. Ben-Asher, J. A. Burns, and E. M. Cliff, "A Brief History of the Time-Optimal Control Problem," IEEE Control Society Magazine, to appear.

The history of time optimal problems is also, to some extent, the history of the optimal control theory since this class of problems marked the way to most important developments in this field. Hermes and LaSalle indicated that there is no other problem, in control theory, about which our knowledge is as complete as in case of the finite dimensional time optimal control problem. Most of this knowledge was developed during the years 1949-1960, in two important centers, the RAND corporation in the US and the Academy of Sciences of the USSR. An important contribution is also credited to LaSalle himself. The time optimal control problem in infinite dimensional space has been investigated ever since 1960 and this process has not been completed yet. We shall try to sketch here the history of the problem examining the main ideas and concepts rather than going into mathematical details. Therefore we shall confine the discussion to contributions of a more general and basic nature, rather than specific problems and applications of time optimal controls.

15. R. S. Chowdhry and E. M. Cliff, "Part II: Time Optimal Rigid Body Motions," Journal of Optimization Theory and Applications, to appear.

Time-optimal control of rigid-body angular rates is investigated in the absence of direct control over one of the angular velocity components. Existence of singular subareas in time-optimal trajectories is explored. Numerical survey of optimality conditions reveals that over a large range of boundary conditions there are, in general, several distinct extremal solutions. A classification of extremal solutions is presented and domains of existence of the extremal sub-families are established in a reduced parameter space. A locus of Darboux points is obtained and global optimality of extremal solutions is observed in relation to Darboux points. Continuous dependence of optimal trajectories with respect to variations in control constraints is noted and a procedure to obtain the time-optimal bang-bang solutions is presented.

16. R. S. Chowdhry and E. M. Cliff, "On Optimal Rigid Body Motions: An Approximate Formulation," Journal of Optimization Theory and Applications, to appear.

Optimal rigid body angular motions are investigated in the absence of direct control over on of the angular velocity components, via an approximate dynamic model. An analysis of first order necessary conditions for optimality with the proposed model reveals that over a large range of boundary conditions there are, in general, several distinct extremal solutions. A classification in terms of sub-families of extremal solutions is presented. Second order necessary conditions are investigated to establish local optimality for the candidate minimizers.

IV. Presentations

1. A. H. Nayfeh, "Can the Practicing Engineer Afford to be Ignorant of Nonlinear Phenomena?," Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, FL, November 18-20, 1987.

2. J. Z. Ben-Asher, J. A. Burns, and E. M. Cliff, "Time-Optimal Slewing of a Flexible Spacecraft," Proceedings of the 26th IEEE Conference on Decision and Control, pp. 524-528, December 9-11, 1987.

3. J. A. Burns, E. M. Cliff, and R. E. Miller, "Control of a Viscoelastic Shaft with Attached Tip Mass," Proceedings of the 26th IEEE Conference on Decision and Control, pp. 997-1001, December 9-11, 1987.

4. A. H. Nayfeh and L. D. Zavodney, "The Response of a Beam with Concentrated Mass to Parametric Excitation," Fifth Annual Review, Virginia Tech Center for Composite Material and Structures, VPI&SU, Blacksburg, VA, April 4-6, 1988.

5. A. H. Nayfeh and J. Nayfeh, "Modal Interactions in the Response of a Beam to a Harmonic Excitation," Fifth Annual Review, Virginia Tech Center for Composite Material and Structures, VPI&SU, Blacksburg, VA, April 4-6, 1988.

6. A. H. Nayfeh and F. Pai, "Complicated Responses of a Cantilevered Beam to a Harmonic Excitation at the Base, Fifth Annual Review, Virginia Tech Center for Composite Material and Structures, VPI&SU, Blacksburg, VA, April 4-6, 1988.

7. A. H. Nayfeh, "Analytical and Experimental Studies of Nonlinear Phenomena," 6th Annual Forum on Space Structures, Atlanta, GA, April 7-8, 1988.

8. A. H. Nayfeh and A. M. A. Hamdan, "Interaction of Nonlinearities and Linear State Feedback," IEEE Conference and Exhibit, Knoxville, TN, April 10-13, 1988.

9. A. H. Nayfeh and L. D. Zavodney, "Parametric Resonances in Nonlinear Structural Elements: Theory and Experiment," ASCE Engineering Mechanics Division Specialty Conference, VPI&SU, Blacksburg, VA, May 23-25, 1988.

10. A. H. Nayfeh and P. F. Pai, "Nonlinear Nonplanar Parametric Responses of an Inextensional Beam," Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

11. A. H. Nayfeh, B. Balachandran, M. A. Colbert, and M. A. Nayfeh, "Theoretical and Experimental Investigation of Complicated Responses of a Two-Degree-of-Freedom Structure", Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

12. A. H. Nayfeh, J. F. Nayfeh, and D. T. Mook, "Modal Interactions in the Response of Beams to a Harmonic Excitation", Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

13. A. H. Nayfeh and L. D. Zavodney, "The Nonlinear Response of a Slender Beam Carrying a Lumped Mass to a Principal Parametric Excitation," Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

14. A. H. Nayfeh and L. D. Zavodney, "Modal Interactions in the Nonlinear Response of Structural Elements- Theory and Experiment," Third

International Conference on Recent Advances in Structural Dynamics, Southampton, England, July 18-22, 1988.

15. A. H. Nayfeh, "Numerical-Perturbation Methods in Mechanics," Symposium on Advances and Trends in Computational Structural Mechanics and Fluid Dynamics, Washington, DC, October 17-19, 1988.

16. J. A. Burns, Z. Y. Liu, and R. E. Miller, "Control of a Thermoviscoelastic System," Proceedings of the 27th IEEE Conference on Decision and Control, pp. 1249-1252, 1988.

17. J. Z. Ben-Asher and E. M. Cliff, "Soft-Constrained Time Optimal Maneuvering of a Flexible Spacecraft," Proceedings of the 30th Israeli Conference on Aviation and Astronautics, pp. 218-225, 1989.

18. R. S. Chowdhry, E. M. Cliff, and F. H. Lutze, "On Optimal Rigid Body Motions," Paper AIAA-89-3616-CP, Proceedings of the AIAA Guidance, Navigation and Control Conference, pp. 1560-1569, 1989.

19. A. H. Nayfeh, "Modal Interactions in Systems with Quadratic Nonlinearities", Pan-American Congress of Applied Mechanics, Sponsored by AAM, Rio de Janeiro, Brazil, January 3-6, 1989. January 29-February 3, 1989.

20. A. H. Nayfeh and B. Balachandran, "Nonlinear Modal Interactions in a Composite Structure," Sixth Annual Review for the Center for Composite Materials and Structures, Blacksburg, VA, April 9-11, 1989.

21. A. H. Nayfeh, J. F. Nayfeh, and D. T. Mook, "Nonlinear Dynamic Response of Laminated Composite Plate Strips in Cylindrical Bending," Sixth Annual Review for the Center for Composite Materials and Structures, Blacksburg, VA, April 9-11, 1989.

22. A. H. Nayfeh and P. F. Pai, "Nonlinear Nonplanar Parametric Responses of an Inextensional Beam," 7th VPI&SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, VA, May 8-10, 1989.

23. A. H. Nayfeh and B. Balachandran, "Modal Interactions in Resonantly Forced Structures," 7th VPI&SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, VA, May 8-10, 1989.

24. A. H. Nayfeh, M. S. El-Zein, and J. F. Nayfeh, "Nonlinear Oscillations of Composite Plates Using Perturbation Techniques," 4th Technical Conference on Composite Materials, VPI&SU, Blacksburg, VA, October 3-6, 1989.

25. A. H. Nayfeh and B. Balachandran, "Nonlinear Vibrations of a Composite Structure," 60th Shock and Vibration Symposium, Virginia Beach, VA, November 14-16, 1989.